OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT

4725/01

MATHEMATICS

Further Pure Mathematics 1

MONDAY 11 JUNE 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The complex number $a + ib$ is denoted by z. Given that $|z| = 4$ and $\arg z = \frac{1}{3}\pi$, find a and b. [4]

2 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$. [5]

3 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(3r^2 - 3r + 1) = n^3.$$ [6]

4 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & 5 \end{pmatrix}$.

(i) Find $\mathbf{A}^{-1}$. [2]

The matrix $\mathbf{B}^{-1}$ is given by $\mathbf{B}^{-1} = \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$.

(ii) Find $(\mathbf{AB})^{-1}$. [4]

5 (i) Show that

$$\frac{1}{r} - \frac{1}{r+1} = \frac{1}{r(r+1)}.$$ [1]

(ii) Hence find an expression, in terms of n, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}.$$ [3]

(iii) Hence find the value of $\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$. [3]

6 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) (a) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. [2]

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(ii) (a) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(b) Use your answer to part (ii)(a) to find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [2]

7 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 0 \\ 0 & a & 4 \\ 2 & 3 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of **M**. [3]

(ii) In the case when $a = 2$, state whether **M** is singular or non-singular, justifying your answer. [2]

(iii) In the case when $a = 4$, determine whether the simultaneous equations

$$\begin{aligned} ax + 4y \quad\quad &= 6, \\ ay + 4z &= 8, \\ 2x + 3y + \ z &= 1, \end{aligned}$$

have any solutions. [3]

8 The loci C_1 and C_2 are given by $|z - 3| = 3$ and $\arg(z - 1) = \frac{1}{4}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [6]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3| \leqslant 3 \text{ and } 0 \leqslant \arg(z - 1) \leqslant \tfrac{1}{4}\pi.$$ [2]

9 **(i)** Write down the matrix, **A**, that represents an enlargement, centre (0, 0), with scale factor $\sqrt{2}$. [1]

(ii) The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by **B**. [3]

(iii) Given that **C** = **AB**, show that $\mathbf{C} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. [1]

(iv) Draw a diagram showing the unit square and its image under the transformation represented by **C**. [2]

(v) Write down the determinant of **C** and explain briefly how this value relates to the transformation represented by **C**. [2]

10 **(i)** Use an algebraic method to find the square roots of the complex number 16 + 30i. [6]

(ii) Use your answers to part **(i)** to solve the equation $z^2 - 2z - (15 + 30\text{i}) = 0$, giving your answers in the form $x + \text{i}y$. [5]

1	*EITHER* $a = 2$ $b = 2\sqrt{3},$ *OR* $a = 2 \quad b = 2\sqrt{3}$	M1 A1 M1 A1 M1 M1 A1 A1	 4 **4**	Use trig to find an expression for a (or b) Obtain correct answer Attempt to find other value Obtain correct answer a.e.f. (Allow 3.46) State 2 equations for a and b Attempt to solve these equations Obtain correct answers a.e.f. SR $\pm$ scores A1 only
2	$(1^3 =)\frac{1}{4} \times 1^2 \times 2^2$ $\frac{1}{4}n^2(n+1)^2 + (n+1)^3$ $\frac{1}{4}(n+1)^2(n+2)^2$	B1 M1 M1(indep) A1 A1	 5 **5**	Show result true for $n = 1$ Add next term to given sum formula Attempt to factorise and simplify Correct expression obtained convincingly Specific statement of induction conclusion
3	$3\Sigma r^2 - 3\Sigma r + \Sigma 1$ $3\Sigma r^2 = \frac{1}{2}n(n+1)(2n+1)$ $3\Sigma r = \frac{3}{2}n(n+1)$ $\Sigma 1 = n$ n^3	M1 A1 A1 A1 M1 A1	 6 **6**	Consider the sum of three separate terms Correct formula stated Correct formula stated Correct term seen Attempt to simplify Obtain given answer correctly
4	(i) $\frac{1}{2}\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}$ (ii) $\frac{1}{2}\begin{pmatrix} 2 & 0 \\ 23 & -5 \end{pmatrix}$	B1 B1 M1 M1(indep) A1ft A1ft	 2 4 **6**	Transpose leading diagonal and negate other diagonal or solve sim. eqns. to get 1st column Divide by the determinant or solve 2nd pair to get 2nd column Attempt to use $B^{-1}A^{-1}$ or find B Attempt at matrix multiplication One element correct, a.e.f, All elements correct, a.e.f. NB ft consistent with their (i)

5	(i) $\frac{1}{r(r+1)}$ (ii) $1 - \frac{1}{n+1}$ (iii) $S_\infty = 1$ $\frac{1}{n+1}$	B1 M1 M1 A1 B1ft M1 A1 c.a.o.	1 3 3 **7**	Show correct process to obtain given result Express terms as differences using (i) Show that terms cancel Obtain correct answer, must be *n* not any other letter State correct value of sum to infinity Ft their (ii) Use sum to infinity – their (ii) Obtain correct answer a.e.f.
6	(i) (a) $\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \gamma\alpha = 2$ (b) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$ $= 9 - 4 = 5$ (ii) (a) $\frac{3}{u^3} - \frac{9}{u^2} + \frac{6}{u} + 2 = 0$ $2u^3 + 6u^2 - 9u + 3 = 0$ (b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -3$	B1 B1 M1 A1 ft M1 A1 M1 A1ft	2 2 2 2 **8**	State correct values State or imply the result and use their values Obtain correct answer Use given substitution to obtain an equation Obtain correct answer Required expression is related to new cubic stated or implied -(their "b" / their "a")

7	(i) $a(a-12)+32$ (ii) det **M** = 12 non-singular (iii) *EITHER* *OR*	M1 M1 A1 M1 A1ft B1 M1 A1 M1 A1 A1	 3 2 3 **8**	Show correct expansion process Show evaluation of a 2 x 2 determinant Obtain correct answer a.e.f. Substitute $a = 2$ in their determinant Obtain correct answer and state a consistent conclusion det M = 0 so non-unique solutions Attempt to solve and obtain 2 inconsistent equations Deduce that there are no solutions Substitute $a = 4$ and attempt to solve Obtain 2 correct inconsistent equations Deduce no solutions
8	(i) Circle, centre (3, 0), y-axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1st quadrant only (ii) Inside circle, below line, above x-axis	B1B1 B1 B1 B1 B1 B2ft	 6 2 **8**	Sketch showing correct features N.B. treat 2 diagrams asa MR Sketch showing correct region SR: B1ft for any 2 correct features

9	(i) $\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$	B1	1	Correct matrix
	(ii) Rotation (centre O), 45^0, clockwise	B1B1B1	3	Sensible alternatives OK, must be a single transformation
	(iii)	B1	1	Matrix multiplication or combination of transformations
	(iv) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	M1 A1	2	For at least two correct images For correct diagram
	(v) det **C** = 2	B1		State correct value
	area of square has been doubled	B1	2	State correct relation a.e.f.
			9	
10	(i)	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ and 16+30i
	$x^2 - y^2 = 16$ and $xy = 15$	A1A1		Obtain each result
		M1		Eliminate to obtain a quadratic in x^2 or y^2
	$\pm(5 + 3i)$	M1		Solve to obtain $x = (\pm)\ 5$ or $y = (\pm)\ 3$
		A1	6	Obtain correct answers as complex numbers
	(ii) $z = 1 \pm \sqrt{16 + 30i}$	M1*		Use quadratic formula or complete the square
	$6 + 3i$, $-4 - 3i$	A1 *M1dep A1 A1ft	5	Simplify to this stage Use answers from (i) Obtain correct answers
			11	